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IMPROVED DFT-MATRIX BASED PILOT ALLOCATION SCHEME FOR SPARSE CHANNEL ESTIMATION IN OFDM SYSTEM USING COMPRESSED SENSING

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Abstract *In this paper, the deterministic pilot allocation problem in orthogonal frequency division multiplexing (OFDM) systems based on mutual coherence minimization of the measurement matrix is addressed using compressed sensing (CS). Pilot symbols are necessary for channel estimation and an improper allocation directly degrades the channel estimation accuracy and significantly impair the performance of the overall system. CS requires that the mutual coherence between the sensing matrix X and the sparsifying basis D (i.e., Discrete Fourier Transform (DFT) submatrix) is small for an optimal pilot pattern allocation. It is known that if the set of pilot pattern is a Cyclic Difference Set (CDS), the mutual coherence of the measurement matrix is minimized. However, CDS in most practical OFDM system is not available. Therefore, a new criterion and an improved DFT-matrix based pilot allocation scheme are proposed that jointly optimized for the allocation of pilots as a solution. CS reconstruction algorithms such as the Orthogonal matching pursuit (OMP), the Regularized OMP (ROMP) and the Subspace Pursuit (SP) are applied in order to recover the sparse channel. Simulation results show that the proposed pilot allocation scheme and criterion offer a better channel estimation performance in terms of BER.*

Key words: *Channel estimation, Communication systems, Compressed sensing, Pilot allocation, Signal processing, Sparse matrices*

1. Introduction

Orthogonal frequency division multiplexing (OFDM) technique has been widely used in the wireless communication system as a result of its high data rate transmission and robustness against the multipath fading channel. The OFDM system technique converts a frequency-selective fading wireless channel into a number of flat-fading narrowband subcarriers [1],[2]. In order to achieve an accurate Channel State Information (CSI) performance, the majority of the wireless communication systems receivers adopt the coherent detection of signals [3]. This is achieved through proper allocation of the pilot symbols [4]. However, in a coherent digital wireless communication system, acquiring accurate estimates of the CSI is critical at the receiver [4]. Studies have shown that the equidistant pilot allocation in the traditional Channel Estimation (CE) approach minimizes the MSE performance but, unfortunately, this does not hold for sparse channels [5].

Recently, channel measurements have shown that the wireless channel exhibits a sparse nature which is due to the sparse structure of scatters in space [6]. Therefore, unlike the traditional CE approach, considering the inherent sparsity of the signal allows for an accurate signal recovery from far fewer measurements to achieve the same MSE performance [7] [4]. Hence, pilot allocation design scheme has become a major focus for sparse CE in OFDM system. According to the Restricted Isometry Property (RIP), high probability of sparse recovery guarantees is now possible and considered as statistically optimal if the observation signal measurement uses matrices that are generated at random. But, there is no

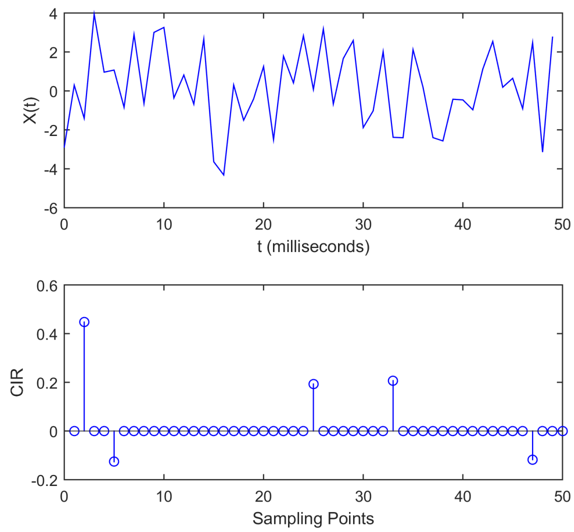

Figure 1. Top: the real part of time-domain signal with 16 measurements or samples. Bottom: 5-sparse vector of Fourier coefficients of length 50

known method to test in polynomial time if a given measurement matrix satisfies RIP [8]. Furthermore, provided with a partial discrete Fourier transform (DFT) measurement matrix, it is known that if the pilot indices set is a cyclic difference set (CDS), the mutual coherence of the measurement matrix is minimized [9][10]. However, it is not guaranteed that a CDS will exist for every pilot size. Alternatively, since the CDS do not exist and it is usual to replace the RIP condition with the mutual coherence measure [4]. Therefore, in this paper, a new criterion that reduces the larger summation of columns correlation of the DFT-submatrix is proposed. Also, a pilot pattern optimization algorithm namely a forward-backward search algorithm is presented for a deterministic pilot allocation design solution.

The remaining part of this paper is organized as follows. The CIR and the OFDM system is presented in Section 2. In Section 3, the proposed criterion and Pilot pattern forward Search allocation algorithm is presented. In Section 4, the simulation results is presented. Chapter 5 concludes the paper.

2 CIR and the OFDM system model

In this section, the CIR model is presented and provides a mathematical framework for the sparsity of high dimensional signals. The OFDM system model is also presented.

2.1 CIR Model

In wireless communication system, the notion of a rich multipath channel structure is normally assumed for the conventional channel estimation method [7] . For high-speed wireless communication system, the CIR is considered to be sparse and consist of a significant number of nonzero dominant channel taps in the entire CIR length. Such channels, an example of which is shown in Fig. 1, typically demonstrates the strength of CS and it presents an illustration of a signal of length $L = 50$, with sparsity $k = 5$ in the Fourier transform domain. The channel is composed of a few dominant taps (i.e., 5 taps) and a large part of taps is approximately zero or zero. This phenomenon, however, is encountered in many physical applications that are of high speed. Consider a frequency-selective multipath fading channel whose CIR is given by

$$h(n) = \sum_{l=0}^{L-1} h_l \delta(n - \tau_l), \tag{1}$$

where $h = [h_0, h_1, \dots, h_L - 1]^T$ is the sparse CIR that must be estimated, h_l is the corresponding complex amplitude of the l^{th} CIR tap, while τ_l is the path delay.

2.2 OFDM system model

Consider an OFDM system with a comb-type pilot arrangement having N subcarriers, of which $\mathcal{M}$ subcarriers are indexed with pilots, in positions: $\mathcal{P} = \{P_1, P_2, \dots, P_{\mathcal{M}}\} \subseteq \{1, \leq P_1 < P_2 < \dots P_{\mathcal{M}} \leq N\}$ that are known at the receiver. From Eq. (1), if the channel is considered to have an L number of sparse channel taps, and if the transmitted pilot vector is represented as $x(P_1), x(P_2), \dots, x(P_{\mathcal{M}})$, then the received pilot vector can be expressed as

$$\begin{bmatrix} y(P_1) \\ y(P_2) \\ \vdots \\ y(P_{\mathcal{M}}) \end{bmatrix} = \begin{bmatrix} x(P_1) & 0 & 0 & 0 \\ 0 & x(P_2) & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & x(P_{\mathcal{M}}) \end{bmatrix} \cdot D_{\mathcal{M}\times L} \begin{bmatrix} h(1) \\ h(2) \\ \vdots \\ h(L) \end{bmatrix} + \begin{bmatrix} v(1) \\ v(2) \\ \vdots \\ v(\mathcal{M}) \end{bmatrix}, \tag{2}$$

where $\boldsymbol{h} \triangleq [h(1), h(2), \dots, h(L)]^T$ is the sparse CIR consisting of only a few number of non-zero channel tap of length L, $\boldsymbol{v} \triangleq [v(1), v(2), ..., v(P_{\mathcal{M}})]^T \sim \mathcal{CN}(0, \sigma_v^2 I_{\mathcal{M}})$ is the additive white Gaussian Noise (AWGN) and the partial DFT submatrix is chosen such that $D_{\mathcal{M}\times L}$ represents the M rows and the first $1, 2..., L$ columns of a standard Fourier transform matrix. The $D_{\mathcal{M}\times L}$ sub-matrix can be expressed as

$$D_{\mathcal{M}xL} = \frac{1}{\sqrt{N}} \cdot \begin{bmatrix} e^{\frac{-j2\pi P_1 \cdot 0}{N}} & e^{\frac{-j2\pi P_1 \cdot 1}{N}} & \cdots & e^{\frac{-j2\pi P_1 \cdot (L-1)}{N}} \\ e^{\frac{-j2\pi P_2 \cdot 0}{N}} & e^{\frac{-j2\pi P_2 \cdot 1}{N}} & \cdots & e^{\frac{-j2\pi P_2 \cdot (L-1)}{N}} \\ \vdots & \vdots & \vdots & \vdots \\ e^{\frac{-j2\pi P_{\mathcal{M}} \cdot 0}{N}} & e^{\frac{-j2\pi P_{\mathcal{M}} \cdot 1}{N}} & \cdots & e^{\frac{-j2\pi P_{\mathcal{M}} \cdot (L-1)}{N}} \end{bmatrix}. \tag{3}$$

If the transmitted pilot signal is denoted as $\mathbf{X} \triangleq diag\{x(P_1), x(P_2), ..., x(P_{\mathcal{M}})\}$, and the received signal is denoted as $\mathbf{y} \triangleq [y(P_1), y(P_2), \dots, y(P_{\mathcal{M}})]^T$ then Eq. (2) can be re-expressed as

$$\mathbf{y} \triangleq \mathbf{X}\, D_{\mathcal{M}XL}\, \boldsymbol{h} + \boldsymbol{v}. \tag{4}$$

If the measurement matrix $\boldsymbol{X} D_{\mathcal{P}\times L}$ is represented by $\mathcal{A}$ then Eq. (4) can be presented as

$$\mathbf{y} \triangleq \mathcal{A}\boldsymbol{h} + \boldsymbol{v}\ . \tag{5}$$

The mathematical model of Eq. (5) directly corresponds to a signal measurement process that is nonadaptive and senses a k-sparse signal h by taking m linear measurements of the signal. Hence, h is reliably recovered from the knowledge of the observation vector signal y and the sensing matrix $\mathcal{A}$. If the measurement matrix $\mathcal{A}$ consist of a higher number of rows than columns ($\mathcal{M} \geq L$) then the CIR can adequately be estimated by LS channel estimation method. However, if the channel consists of a lesser number of rows than columns ($\mathcal{M} < L$) then the system is said to be under-determined and can no longer be estimated by LS. In this case, CS algorithm can be applied in estimating $\boldsymbol{h}$ with a high degree of accuracy. Since the number of rows (i.e., pilot signals) is far lesser than the number of columns (i.e., channel coefficients), the pilot signal overhead typically experiences a reduction and result in the conservation of spectral efficiency. In this work, in order to conserve spectral efficiency, the $\mathcal{M} < L$ case is considered.

2.3 Sparse Channel Estimation

Considering the case of $\mathcal{M} < L$, three different questions now arise, (i) To achieve a successful recovery guarantee of the sparse CIR h, what specific conditions must the measurement matrix $\mathcal{A}$ satisfy in order to guarantee an accurate recovery of the signals? (ii) From the observation signal y and the measurement matrix $\mathcal{A}$ in Eq. (5), can the CIR, h in practice be dependently solved using the list of the available Polynomial-time algorithms? and (iii) In the event that the observation signals y is degraded by noise perturbation, what guarantee in terms of performance can be reasonably rendered?

Solution to these questions have over the years been given by a lot of researchers mostly in the field of compressed Sensing (CS) which stresses that the $k-$sparse or approximately $k-$sparse CIR h can efficiently be recovered from the noisy measurement y by using both a well-designed measurement matrix $\mathcal{A}$ of dimension $\mathcal{M} < L$ and a good non-linear reconstruction algorithm [11]. This result rely on the fact that, the Restricted Isometry Property sees every $2k$ column of the normalized measurement matrix $\mathcal{A}$, behaving as an isometry. The RIP is defined as follows

Definition 1:[11] A measurement matrix $\mathcal{A}$ satisfies the RIP of order k if there exist a $\delta_k \in (0,1)$ such that

$$(1-\delta_k)\|h\|_2^2 \leqslant \|Ah\|_2^2 \leqslant (1+\delta_k)\|h\|_2^2 \tag{6}$$

holds for all $\boldsymbol{h} \in \sum_k$. To put it in another way, the RIP of order $2k$ emphasizes that the matrix $\mathcal{A}$ behaves in an almost isometry on all the k sparse vector $\boldsymbol{h}$. For instance, if $\mathcal{A}$ is an identity matrix of order L, RIP is then trivially satisfied for any $k = 1, 2, ..., L$ having $\delta_k = 0$. Hence, $\mathcal{A}$ verifiable satisfies these properties but with a combinatorial computational complexity of submatrices that must essentially be considered in guaranteeing recovery. Therefore, other related measures on the matrix $\mathcal{A}$, such as mutual coherence, are often used and its defined as follows

Definition 2:[4] The coherence of the measurement matrix $\mathcal{A}$ is defined as the largest absolute inner product correlation between any two separate columns of $\mathcal{A}$ and is presented as

$$\begin{aligned}
\mu(\mathcal{A}) &= \max_{0\leq m<n\leq L-1} |\langle \mathcal{A}(m), \mathcal{A}(n)\rangle| \\
&= \max_{0\leq m<n\leq L-1} |(\mathcal{A}^H\mathcal{A})_{mn}| \\
&= \max_{0\leq m<n\leq L-1} \left| \sum_{i=1}^{\mathcal{M}} X(P_i)e^{-j2\pi(-P_i m)/N} X(P_i)e^{-j2\pi P_i n/N} \right| \\
&= \max_{0\leq m<n\leq L-1} \left| \sum_{i=1}^{\mathcal{M}} |X(P_i)|^2 e^{-j2\pi P_i(n-m)/N} \right|.
\end{aligned} \tag{7}$$

Tropp and Gilbert [1] have shown that, the Basis pursuit (BP) and the simple Orthogonal Matching Pursuit (OMP) algorithms can accurately reconstruct the originally transmitted signal, if only the following theorem is satisfied.

Theorem 1:[1] Suppose μ is the coherence of A, $\mu(\mathcal{A})$ and that $h \in \sum_k$ with $k < (1/\mu+1)/4$. In addition, suppose $\boldsymbol{y}$ measurements were obtained from the observation signal i.e. $\boldsymbol{y} = \mathcal{A}\boldsymbol{h} + \boldsymbol{v}$, then when $\mathcal{B}(\boldsymbol{y}) = \{\boldsymbol{h} : \|\mathcal{A}\boldsymbol{h} - \boldsymbol{y}\|_2 \leqslant \epsilon$, the reconstructed signal deviation of $\hat{\boldsymbol{h}}$ from $\boldsymbol{h}$ using BP and OMP is bounded by

$$\|\hat{\boldsymbol{h}} - \boldsymbol{h}\|_2 \leqslant \frac{\epsilon^2}{\sqrt{1-\mu(\mathcal{A})(2k-1)}}, \tag{8}$$

where $\mathcal{B}(y)$ ensures that the estimate of $\hat{\boldsymbol{h}}$ is consistent with the observed measurements $\boldsymbol{y}$. Since $\mu(\mathcal{A})$ is the only variable property in Eq. (8), therefore means that, a measurement matrix designed in such a way that $\mu(\mathcal{A})$ is the least possible will proffer a more accurate approximation of h.

In signal processing, it is known that, a complex-valued signal has a constant amplitude zero autocorrelation waveform property with modulus one and with zero autocorrelation function. Since, OFDM system pilot pattern sequences used in wireless communication exhibit this property, the pilot patterns in this paper will be assumed to have a constant amplitude of one ($|X(P_1)| = |X(P_2)| \ldots |X(P_{\mathcal{M}})| = 1$). In this case, according to Eq. (7), for every set $\mathcal{P}$, the coherence will be computed based on just the effect of pilots across the P_i location where $i \in 1, 2, \ldots \mathcal{M}$, while the pilot amplitudes are relaxed to a value of one based on the stated assumption. The Eq. (7) is further simplified as;

$$\mu(A) = 1 \cdot \max_{0\leq m<n\leq L-1} \left| \sum_{i=1}^{\mathcal{M}} e^{-j2\pi K_i(n-m)/N} \right|. \tag{9}$$

If $\Lambda(\mathcal{P}) = \mu(\mathcal{A})$, the objective function is to select the pilot pattern that has the minimum argument of the mutual coherence measure, this is presented as

$$\mathcal{P}_{opt} = arg \min_{\mathcal{P}} \Lambda(\mathcal{P}). \tag{10}$$

2.4 Mutual Coherence Analysis

In Eq. (9), the coherence of the measurement matrix $\mu(\mathcal{A})$ only relies on $q = n - m$ (i.e., the pilot location). Therefore, Eq. (9) can further be re-expressed as

$$\mu(A) = 1 \cdot \max_{0 \leq q \leq L-1} \left| \sum_{i=1}^{\mathcal{M}} e^{-j2\pi K_i q/N} \right| \tag{11}$$

Consequently, the optimal pilot pattern solution is thus derived as;

$$\begin{aligned}
\mathcal{P}_{opt} &= arg \min_{\mathcal{P}} \max_{0 \leq q \leq L-1} \left| \sum_{i=1}^{\mathcal{M}} e^{-j2\pi P_i q/N} \right| \\
&= arg \min_{\mathcal{P}} \max_{0 \leq q \leq L-1} \left\| \sum_{i=1}^{\mathcal{M}} e^{-j2\pi P_i q/N} \right\|^2 \\
&= arg \min_{\mathcal{P}} \max_{0 \leq q \leq L-1} \left(\sum_{i=1}^{\mathcal{M}} e^{-j2\pi P_i q/N} \right) \left(\sum_{i=1}^{\mathcal{P}} e^{-j2\pi P_i q/N} \right)^H \\
&= arg \min_{\mathcal{P}} \max_{0 \leq q \leq L-1} \left(\sum_{i=1}^{\mathcal{M}} e^{-j2\pi P_i q/N} \right) \left(1 \Big/ \sum_{i=1}^{\mathcal{P}} e^{-j2\pi P_i q/N} \right) \\
&= arg \min_{\mathcal{P}} \max_{0 \leq q \leq L-1} \left(\sum_{i=1}^{\mathcal{M}} e^{-j2\pi P_i q/N} \right) \left(\sum_{i=1}^{\mathcal{P}} e^{j2\pi P_i q/N} \right) \\
&= arg \min_{\mathcal{P}} \max_{0 \leq q \leq L-1} \sum_{m=1}^{\mathcal{M}} \sum_{n=1}^{\mathcal{M}} e^{-j2\pi q (P_m - P_n)/N}.
\end{aligned} \tag{12}$$

Let $\mathcal{C}_{\mathcal{P}}$ represents the cyclic difference set of $\mathcal{P}$, such that

$$\mathcal{C}_{\mathcal{P}} = \{(P_n - P_m) \quad mod \;\; N | 1 \leq m \neq n \leq \mathcal{P}\}. \tag{13}$$

For an element, say d contained in $\mathcal{C}_{\mathcal{P}}$, if a_d represents the number of repetition in $\mathcal{C}_{\mathcal{P}}$, then Eq. (12) can therefore be re-expressed as

$$arg \min_{\mathcal{P}} \max_{0 \leq q \leq L-1} \sum_{m=1}^{\mathcal{M}} \sum_{n=1}^{\mathcal{M}} e^{-j2\pi q (P_m - P_n)/N} = \mathcal{M} + \sum_{d=1}^{N-1} a_d e^{-j2\pi q d/N}. \tag{14}$$

To obtain the Fourier series coefficient sequence of a_d, the complex orthogonal sequence set is exploited. This is achieved by multiplying through by $e^{-j2\pi q n/N}$ and summing from $n = 0$ to $n = N - 1$. This, therefore, gives

$$\sum_{q=1}^{N-1} \left(\mathcal{M} + \sum_{d=1}^{N-1} a_d e^{-j2\pi q d/N} \right) = \mathcal{M}(N-1) - \sum_{d=1}^{N-1} a_d = \mathcal{M}(N - \mathcal{P}). \tag{15}$$

The maximum value of the R.H.S of Eq. (14) within the range of $n = 0$ to $n = N - 1$, can therefore be expressed as

$$\max_{0 \leq q \leq N-1} \left(\mathcal{M}(N-1) - \sum_{d=1}^{N-1} a_d \right) \geq \mathcal{M}(N - \mathcal{M})/(N-1). \tag{16}$$

Hence, this is valid only for an index set of pilot pattern $\mathcal{P}$ where $a_1 = a_2 = ... = a_{N-1} = (\sum_{d=1}^{N-1} a_d)/(N-1) = \mathcal{M}(\mathcal{M}-1)/(N-1) = \lambda$ occurs. Choices for which the measurement matrix coherence is minimized is thus present. For instance, a CDS $\{1, 6, 7, 9, 19, 38, 42, 49\}$ represented as CDS $(57, 8)$ where $N = 57$ and $\mathcal{M} = 8$, satisfies the integer $F(1 \leq F \leq N-1)$ i.e $F(1 \leq F \leq 56)$ and occurs exactly $(\lambda = \mathcal{M}(\mathcal{M}-1)/(N-1) = 8(8-1)/(57-1) = 1)$ 1 times in the set $\Psi \triangleq \{((a_m - a_n))_{57} | 0 \leq m \neq n \leq 7\}$ to which Ψ has 56 entries. Also, a CDS $\{0, 1, 3, 5, 9, 15, 22, 25, 26, 27, 34, 35, 38\}$ represented as CDS (40,13) where $N = 40$ and $\mathcal{M} = 13$, which satisfies the integer $F(1 \leq F \leq N-1)$ occurs exactly 4 times in the set $\Psi \triangleq \{((a_m - a_n))_{40} | 0 \leq m \neq n \leq 12\}$ to which Ψ has 156 entries.

Pilot pattern selection in relation to CDS $(N, \mathcal{M})$ is optimal if the CDS exist [12]. Hence, the mutual coherence of the partial DFT sub-matrix will be minimized. However, in practical OFDM system, the design consideration for N is often of the order of $N = 2^n$, where n is chosen to be $6, 8, 9$, or 10 where the CDS do not exist. The mutual coherence property would therefore be applied for a suboptimal pilot pattern design.

3. Proposed Criterion and Foward Search Pilot Allocation Scheme

In this section, a new criterion and a cost function which is based on the mutual coherence minimization of the DFT submatrix is proposed for the deterministic pilot pattern selection in cases where the CDS does not exist. From Eq. (9), the minimum coherence of the measurement matrix can also be represented having the mathematical exactness as

$$\mu_{max}(\mathcal{A}) = \max_{0 \leq m < n \leq L-1, m \neq n} |\mathcal{G}_{m,n}| = \max_{0 \leq m < n \leq L-1, m \neq n} |\hat{a}_m^H \hat{a}_n|, \tag{17}$$

where $(\cdot)^H$ is the Hermitian transpose, $\hat{a}_m$ represents the m^{th} column of $\mathcal{A}$ and the gram matrix represented as $\mathcal{G}$. It is known that, the average minimum coherence of the off-diagonal entry of $\mathcal{A}$ is given by [9]

$$\mu_{avg}(\mathcal{A}) = \frac{\sum\limits_{0 \leq m < n \leq L-1, m \neq n} |\hat{a}_m^H \hat{a}_n|}{(L-1)[(L-1)-1]}. \tag{18}$$

Therefore, from Eq. (17) and Eq. (18), since the value of coherence lies between $0 < \mu \leq 1$, it is then obvious that

$$\sqrt[\beta]{\frac{\sum\limits_{0 \leq m < n \leq L-1, m \neq n} |\hat{a}_m^H \hat{a}_n|^\beta}{(L-1)[(L-1)-1]}} \leq \mu_{max}(\mathcal{A}). \tag{19}$$

Suppose that δ_S is the infinimum of the Restricted Isometry Constant (RIC) that satisfies the definition of the Restricted Isometry Property (RIP). Therefore, Eq. (19) can be re-expressed as

$$0 \leq \delta_S \leq \sqrt[\beta]{\frac{\sum\limits_{0 \leq m < n \leq L-1, m \neq n} |\hat{a}_m^H \hat{a}_n|^\beta}{(L-1)[(L-1)-1]}} \leq \mu_{max}(\mathcal{A}) \tag{20}$$

with a cost function of $\beta = 4$ as proposed, translates to be the fourth root of the mutual coherence function and which directly corresponds to the upper bound of the RIC constant δ_S. This cost function will be applied while calculating the mutual coherence of the measurement matrix $\mathcal{A}$ which will be suitable in minimizing the summation of columns correlation in the partial DFT submatrix.

Algorithm 1 describes the construction of the matrix $\mathcal{A}$ using an optimal pilot patten selection. The essential idea in the proposed scheme is to generate a series of candidate pilot patterns, where each new candidate is acquired from the previous one by taking a step in the right direction towards the global optimum. Algorithm 1 starts with the initialization process and with a random pilot generation $\mathcal{P} = \{P_1, P_2, \ldots, P_\mathcal{M}\} \subseteq \{1, \leq P_1 < P_2 < \ldots P_\mathcal{M} \leq N\}$ having a cardinality of $\mathcal{P}$. For each index set obtained, the partial DFT sub-Matrix is determined and the mutual coherence of the measurement matrix is calculated. The index set derived from the minimum coherence of $\mathcal{A}$ is constantly updated to form the set $\hat{\mathcal{P}} = \{P_1, P_2, \ldots, P_\mathcal{M}\}$. The process is then repeated in a backwards fashion in order to avoid the proposed algorithm from moving away from its global objective.

Data: $\mathcal{M}$ = number of pilots, N = number of subcarriers, $F_{\mathcal{M}\times L}$ = DFT submatrix
initialization: $Q \Longleftarrow 1$;
$\mathcal{P} = \{P_1, P_2, \ldots, P_{\mathcal{M}}\} \subseteq \{1, \leq P_1 < P_2 < \ldots P_{\mathcal{M}} \leq N\}$ ◁ Randomly generate a pilot pattern
$\hat{\mathcal{P}} \Leftarrow \mathcal{P}$
for *k=1,2,...,J* **do**
  for *m=1,2,...,$\mathcal{M}$* **do**
    for *n=1,2,...,N* **do**
      if $P_m \notin n$ **then**
        for each index set obtain the DFT submatrix $A(= F_{\mathcal{M}\times L})$
        $\mu_{max}(\mathcal{A}) = \max_{0\leq m<n\leq L-1, m\neq n} |\mathcal{G}_{m,n}|$ ◁ compute mutual coherence ;
        if $coherence < Q$ **then**
          Choose substitute for $P_m \setminus n$ with the minimum coherence index from the set
        end
      end
    end
  end
  $\hat{\mathcal{P}} = \{P_1, P_2, \ldots, P_{\mathcal{M}}\}$
  for $p = \mathcal{M} - 1, \mathcal{M} - 2, \ldots, 1$ **do**
    for *n=1,2,...,N* **do**
      if $P_m \notin n$ **then**
        for each index set obtain the DFT submatrix $A(= F_{\mathcal{M}\times L})$
        $\mu_{max}(\mathcal{A}) = \max_{0\leq m<n\leq L-1, m\neq n} |\mathcal{G}_{m,n}|$ ◁ compute mutual coherence ;
        if $coherence < Q$ **then**
          Choose substitute for $P_m \setminus n$ with the minimum coherence index from the set
        end
      end
    end
  end
end

Algorithm 1: Forward Backward Pilot Search Algorithms

4 Simulation Results

Consider a 4QAM OFDM system with $N = 256$ subcarriers and $M = 16$ pilot subcarriers used for OFDM channel estimation. If the channel model is based on an FIR filter with $L = 50$, out of which $k = 5$ are non-zero randomly selected channel taps with varying fading parameters, making the channel $5-$sparse. Pilot allocation design numbers according to presented methods in [4], [9] and our proposed values have been tabulated in Table 1 for comparison. As to the problem of pilot placement, the equidistant pilot allocation is generally optimal in conventional CE such as the Least Squares (LS) estimator which happens to be the best linear unbiased estimator. But, provides inaccurate estimates for sparse CE which requires using randomly chosen sets of pilot signals. The LSE is thus, considered for comparison. In

Table 1. Pilot pattern design by different methods

Method	Optimal Pilot Pattern
LSE	1, 18, 35, 52, 69, 86, 103, 120, 137, 154, 171, 188, 205, 222, 239, 256
[4]	8, 40, 48, 52, 72, 82, 99, 142, 145, 154, 158, 161, 183, 209, 212, 230
[9]	7, 17, 20, 29, 33, 79, 85, 137, 156, 159, 165, 174, 178, 202, 206, 239
Proposed	11, 32, 43, 61, 65, 68, 72, 82, 96, 128, 141, 172, 183, 209, 218, 227

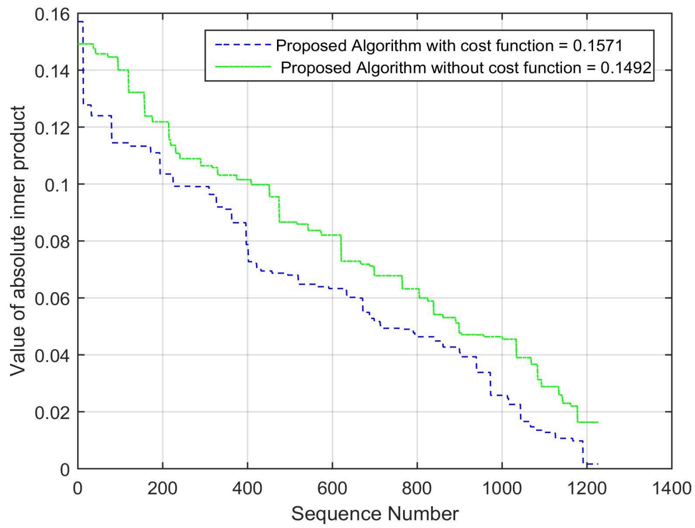

Figure 2. Magnitude in the off-diagonal elements in the measurement matrix for two different methods: Algorithm 1 using the cost function and Algorithm 1 without the cost function

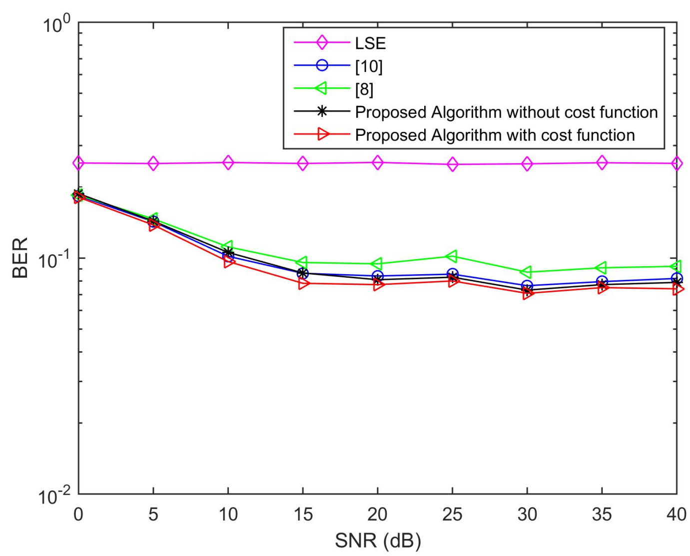

Figure 3. Channel estimation comparison showing BER performance for different optimal pilot pattern using OMP

Fig. 2, the magnitude of the upper triangular of the off-diagonal entries of the Gramian matrix representing the column correlation for two different scenarios of the pilot pattern of the proposed algorithm is depicted. It is observed that, the application of the cost function to the proposed algorithm helps in the further reduction of the magnitude of column correlation of the off- diagonal entries of the measurement matrix (i.e., making the sensing basis more incoherent with the sparsity basis). This is important because, in the CS framework ($y = \mathcal{A}h + v$, where $\mathcal{A} = XD$), the sparsfying discrete Fourier transform basis D should be maximally incoherent with the sensing matrix X for a more accurate guarantee of

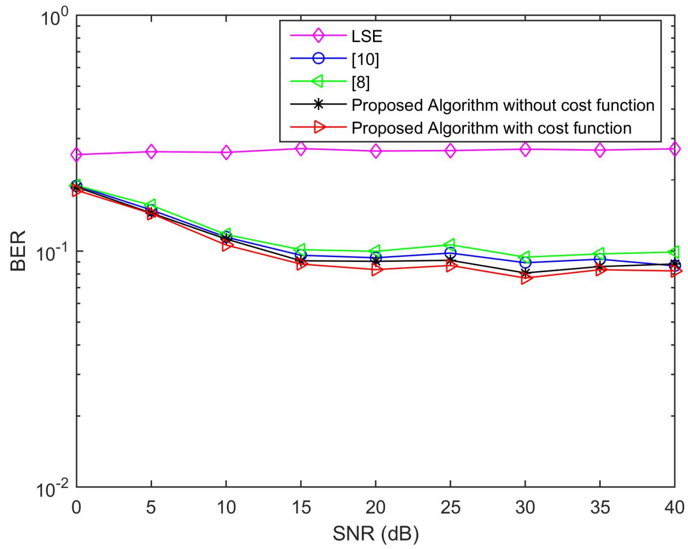

Figure 4. Channel estimation comparison showing BER performance for different optimal pilot pattern using ROMP

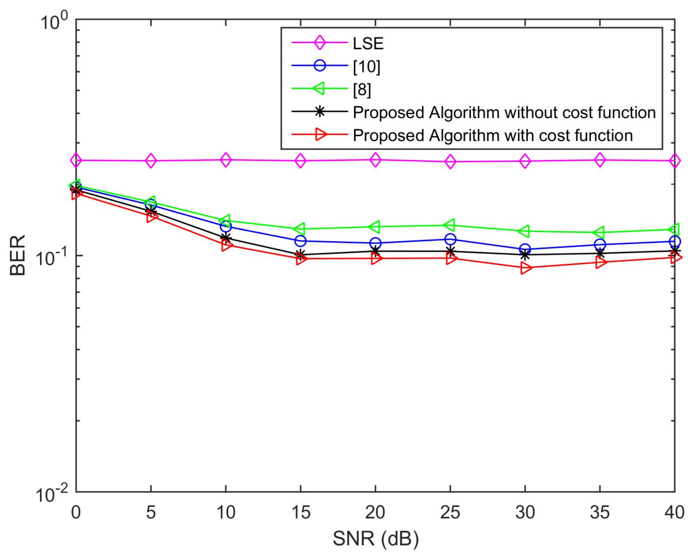

Figure 5. Channel estimation comparison showing BER performance for different optimal pilot pattern using SP

the recovered signal.

Furthermore, sparse channel estimation using the Orthogonal Matching Pursuit (OMP), the Randomized OMP (ROMP) and the Subspace Pursuit (SP) algorithms are employed to estimate the sparse channel. The LS channel estimator which uses the equidistant pilot pattern is also employed in estimating the sparse channel. The pilot pattern obtained from the proposed algorithm and those obtained by different optimal methods as presented in Table 1 are individually used as pilot signals in the OFDM system to track the Channel Impulse Response (CSI) for subsequent sparse signal reconstruction.

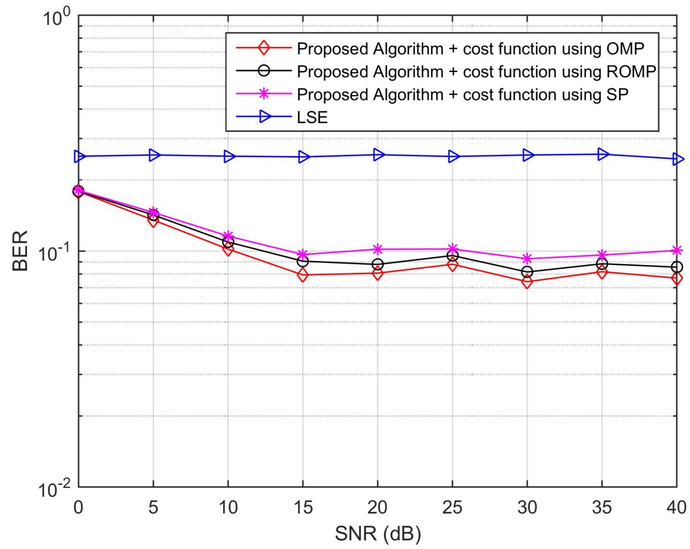

Figure 6. BER recovery performance of the proposed algorithm using the OMP, ROMP, SP compressed sensing estimators on the same curve

The channel estimation performance are compared showing the BER performance for different optimal pilot patterns using the OMP, Randomized OMP (ROMP) and the Subspace Pursuit (SP) Algorithm and the results are presented in Fig. 3, Fig. 4, and Fig. 5 respectively. It is observed that the pilot pattern optimization algorithm obtained from the proposed pilot pattern with the cost function performs better than that without the cost function. Also, it is observed that the proposed method outperforms the other optimized pilot pattern from different schemes. In Fig. 6, it is observed that using the OMP sparse reconstruction algorithm produces a better reconstruction performance than the ROMP and SP algorithms respectively. The reconstruction performance of the LS channel estimation using the equidistant pilot pattern produces a poor reconstruction performance. This is because, the conventional method of channel estimation that employs the least-squares (LS) fail to exploit the wireless channel sparsity, which results in a poor BER performance in a sparse scenario.

5. Conclusion

Compressed Sensing (CS) is a new field of applied mathematics that tackles the drawbacks of conventional signal processing technique, that aims at sampling a signal to efficiently acquire the signal in a compressed form and reconstructing the signal, by finding solutions to the underdetermined linear systems. In this paper, the investigation on the possibility of improving the sparsity-based OFDM system channel estimates was formulated as a pilot pattern allocation problem. First, a new criterion and a fourth power summation of columns correlation were proposed to decrease the size of the larger magnitude column correlation of the DFT sub-matrix since the mutual coherence property is the upper bound of the infinimum restricted isometric constant in RIP. For the cases where the CDS does not exist, a forward-backwards scheme that optimizes pilot pattern allocation was proposed. It was demonstrated that using the compressed sensing technique, the proposed cost function and pilot pattern allocation algorithm leads to better performance in channel estimation. Results from the simulation were confirmed when the Orthogonal Matching Pursuit (OMP) algorithm and the Regularized Orthogonal Matching Pursuit (ROMP) compressed sensing estimators were used for channel estimation.

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References

[1] Joel A Tropp and Anna C Gilbert. Signal recovery from random measurements via orthogonal matching pursuit. *IEEE Transactions on information theory*, 53(12):4655–4666, 2007.

[2] Anthony N Uwaechia and Nor M Mahyuddin. A review on sparse channel estimation in ofdm system using compressed sensing. *IETE Technical Review*, pages 1–18, 2016.

[3] Mingchao Yu and Parastoo Sadeghi. A study of pilot-assisted ofdm channel estimation methods with improvements for dvb-t2. *IEEE Transactions on Vehicular Technology*, 61(5):2400–2405, 2012.

[4] Chenhao Qi, Guosen Yue, Lenan Wu, Yongming Huang, and Arumugam Nallanathan. Pilot design schemes for sparse channel estimation in ofdm systems. *IEEE Transactions on Vehicular Technology*, 64(4):1493–1505, 2015.

[5] Seyyed Hadi Seyyed Masoumian and Behzad Mozaffari Tazehkand. Greedy deterministic pilot pattern algorithms for ofdm sparse channel estimation. *Wireless Personal Communications*, 84(2):1119–1132, 2015.

[6] Wenbo Ding, Fang Yang, Wei Dai, and Jian Song. Time–frequency joint sparse channel estimation for mimo-ofdm systems. *IEEE Communications Letters*, 19(1):58–61, 2015.

[7] Han Wang, Qing Guo, Gengxin Zhang, Guangxia Li, and Wei Xiang. Pilot pattern optimization for sparse channel estimation in ofdm systems. *IEEE Communications Letters*, 19(7):1233–1236, 2015.

[8] Guan Gui, Li Xu, Wentao Ma, and Badong Chen. Robust adaptive sparse channel estimation in the presence of impulsive noises. In *2015 IEEE International Conference on Digital Signal Processing (DSP)*, pages 628–632. IEEE, 2015.

[9] Abbas Kamali, Masoud Reza Aghabozorgi Sahaf, Ali Mohammad Doost Hosseini, and Ali Akbar Tadaion. A low complexity dft-matrix based pilot allocation algorithm for sparse channel estimation in ofdm systems. *AEU-International Journal of Electronics and Communications*, 68(2):85–89, 2014.

[10] Peifang Gao, Yuan Bai, Xiurong Ma, and Bingxue Chen. A novel method on pilot selection for sparse channel estimation in ofdm systems. *IEICE Electronics Express*, 12(10):20150249–20150249, 2015.

[11] Waheed U Bajwa, Akbar M Sayeed, and Robert Nowak. A restricted isometry property for structurally-subsampled unitary matrices. In *47th Annual Allerton Conference on Communication, Control, and Computing, Allerton*, pages 1005–1012. IEEE, 2009.

[12] V Nagendra Babu, V Adinarayana, K Muralikrishna, and P Rajesh Kumar. An improved compressed sensing based sparse channel estimation for mimo-ofdm systems with an efficient pilot insertion scheme. *Indian Journal of Science and Technology*, 9(20), 2016.